# Technical Notes

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## Crossflow Transport Induced by Vortices

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THE three experimental results to be described here are all independently obtained, but they display a common feature: vortical structures induce vigorous crossflow current to such a degree that in a flowing apparatus, which may be presumed to produce reasonably two-dimensional flow, strong three-dimensonality appears. The reason is as follows. The cores of vortices, with their lower pressure, lift and draw the fluid out of the side-wall boundary layer of the apparatus, in a manner quite similar to that of the updraft induced by a tornado near the ground.

The phenomenon first came to our attention through a study of large-scale structures in a shear flow, followed by the similar observations obtained for the von Kármán vortex street. Figure 1 shows a shear layer (a plan view) formed between two streams of air at 160 and 300 cm/s separated by a movable splitter plate. The layer is perturbed under conditions of a two-dimensional sinusoidal oscillation at 115 Hz, corresponding to the Kelvin-Helmholtz inviscid instability. The flow is from left to right, and the end of the splitter plate is slightly to the left, off the picture. The width of the flow is 10 cm. Smoke is introduced on both sides of the splitter plate and is used to view the flow at five spanwise locations. The large-scale coherent structures show up through accumulation of smoke and show their transverse nature in the photograph. Of particular notice are the tornado-like structures at the end of each of the main coherent structures. This appears to be the result of a suction inflow from the wall boundary layer. For spanwise locations closer to the centerline, the apparent transverse flow is smaller and disappears altogether at the centerline. Thus, two-dimensional flow only seems to exist in a triangular region at the beginning of the flowfield, even though the unit Reynolds number of the flow is relatively small. This transverse flow was also seen and studied by Koochesfahani,[1] although in wake flows of airfoils rather than in a shear layer (see also Ref. 2).

We now turn to the von Kármán vortex street. In Figs. 2a and 2b, a cylinder of 1.27 cm diam is placed in a water tunnel, spanning two sidewalls; the flow is from left to right, the free-stream velocity being uniform and equal to 4.57 cm/s. (The Reynolds number based on the diameter is 467.) The centerline of the tunnel is marked by a dark solid line drawn on the bottom, quarter-span lines by broken lines, eighth-span lines by chain lines. In Fig. 2a, where a dye is injected at the leeside and near a sidewall, the presence of a strong spanwise cross-current is clearly visible. Its direction always remains away from the wall. Observe in particular the hooked shape of a darker filament near the wall, which indicates a change from the initial streamwise convection to the lateral transport, carried presumably through a vortex core.

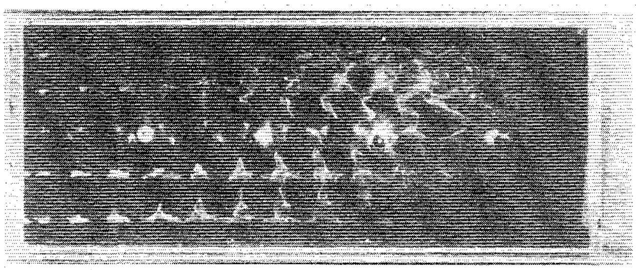

Fig. 1 Crossflow transport in a shear flow.

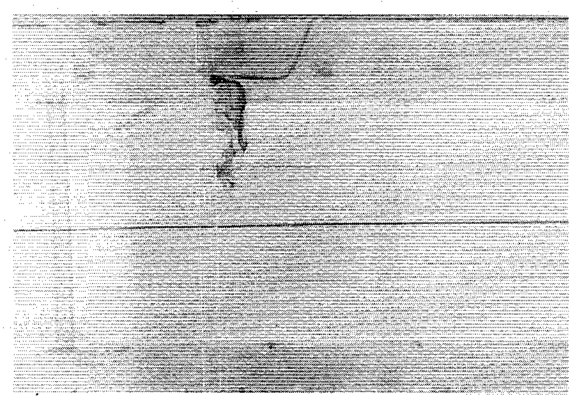

Fig. 2a Crossflow transport in a von Kármán vortex street; dye injected near a sidewall.

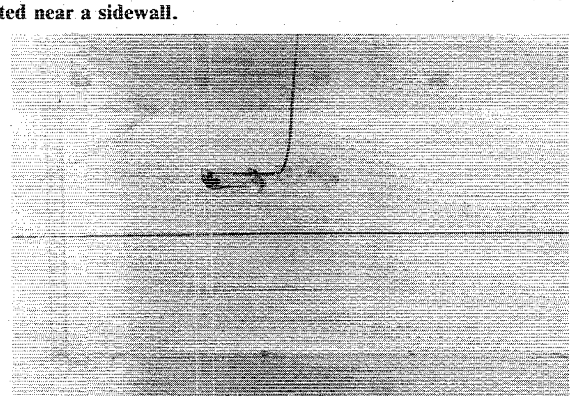

Fig. 2b Dye injected near a quarter-span point.

Received Feb. 22, 1988; revision received June 10, 1988. 

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Because of the symmetry, two spanwise currents are induced from each wall. They flow counter to each other, toward the midspan, where they collide. Hence, the strength of the cross-currents becomes weakened toward the midspan, as observed from Fig. 2b, where a dye is injected near a quarter-span point. A dye, when injected at a midspan, is observed to be washed downstream without showing movement in the spanwise direction.

The overall pattern shown in Fig. 3, taken at the same condition, now exhibits the long-familiar vortex street; here dye is released from a number of ports on the cylinder, rather than the aforementioned off-cylinder positions. This conventional means of dye release tends, however, to conceal the strong spanwise transport, although even in Fig. 3 its vestige is not unidentifiable.

In order to quantify this crossflow transport, a gas tracer experiment was conducted in a wind tunnel. Six ports, arranged in a line parallel to the cylinder axis, are provided on the leeside of a cylinder of 1/2 in. diam; each port is connected to outside via a hyperdermic tubing embedded on the cylinder (Fig. 4). $CO_2$ was chosen as a tracer gas. It was supplied from a bottle, fed to an injection port on the cylinder. A mixture of $CO_2$ and air was drawn by a vacuum pump from a suction port and sampled by a gas analyzer (Infrared Industries, Model No. 702). Digital outputs were processed by a data aquisition system.

Care was taken to limit the flow of $CO_2$ gas so that the injection and suction of the tracer itself would not set up any spurious crossflow. Also, in order to avoid the inadvertent intensification of the vortex strength (hence, the unnatural spanwise transport) at tunnel acoustic resonance,[3] the interior walls of the tunnel were covered with acoustic foam of 2.54-cm thickness. In a preceding experiment, it was confirmed that acoustic resonance at the so-called lock-in Mach numbers[3] could be averted by this means of sound absorption. In the test, the freestream Mach number $M_\infty$, uniform upstream of the cylinder, was varied from 0 to 0.5. (The Reynolds number at $M_\infty = 0.5$ is equal to $1.5 \times 10^5$.)

Figures 4a–4c present the measured $CO_2$ concentration plotted against the freestream Mach number. In each figure two results are shown, both corresponding to two adjacent ports. For instance, at the top of Fig. 4a, suction *s* was applied at a port nearest to a sidewall, the injection *i* at an adjacent port. At the bottom, the role of suction and injection was reversed. The presence of spanwise transport, in a direction away from the wall, is obvious. In the subsequent Figs. 4b and 4c, the positions of the injection/suction ports were progressively moved toward the centerline of the wind tunnel; the opposing influence of two competing currents, both generated by two sidewalls, is self-evident.

The Mach number dependence observable in Fig. 4 appears to evidence a competition between the secondary spanwise transport and the primary steamwise flow. In contrast to lower Mach numbers, where the former is doubtless predominant, at a higher Mach number the latter appears to prevail, where the measured $CO_2$ concentration may simply be due to turbulent diffusion. Although we have not carried out any extensive investigations, the conflict between the two competing factors and their influence are evident in the following observation. In the water tunnel used for the flow visualization, we

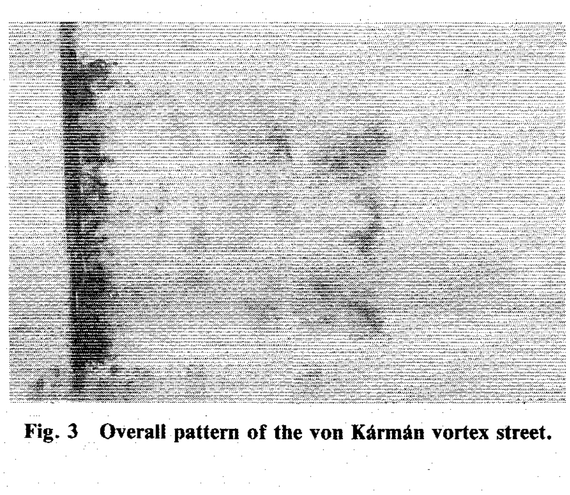

Fig. 3 Overall pattern of the von Kármán vortex street.

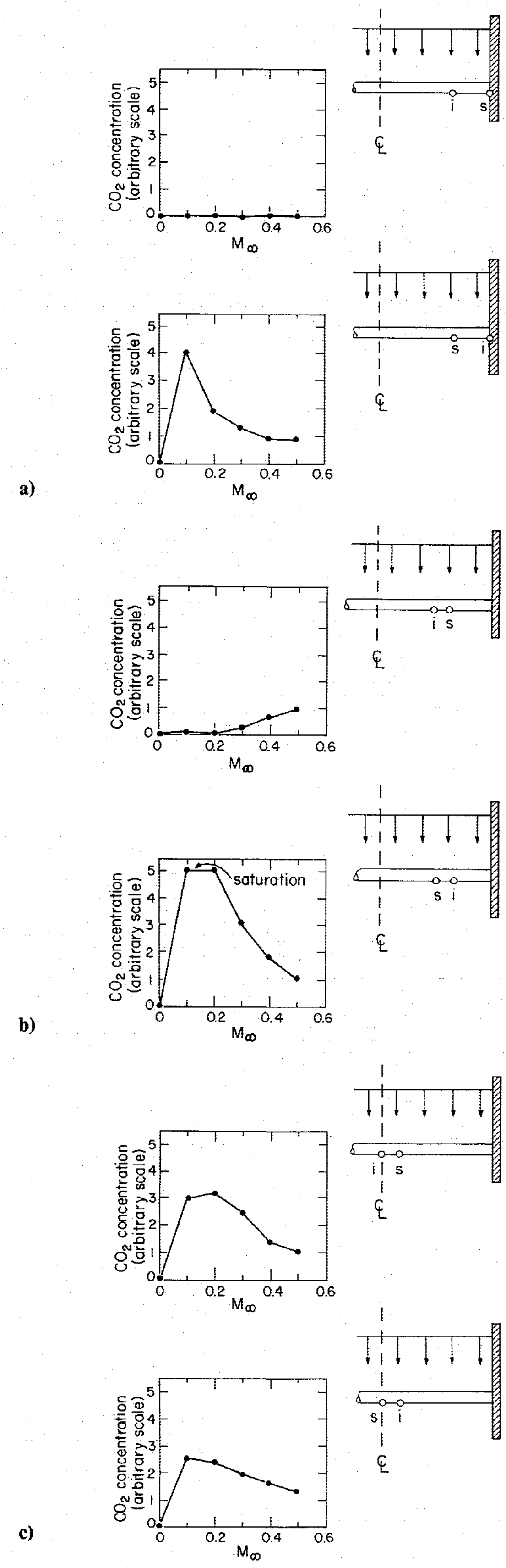


**Fig. 4 Results of gas tracer experiment.**

replaced the circular cylinder by an equilateral triangular one, with one of its flat surfaces facing upstream and an edge pointing downstream. Then the spanwise crossflow is found to disappear; the streamwise acceleration around the two sharp edges of the triangle apparently washes the vortices downstream too rapidly to allow the formation of the secondary spanwise transport.

With regard to the fluid-dynamical details, cross-transport through the vortex core appears to stem from two sources: 1) the solenoidal property of vorticity, and 2) the no-slip condition on a solid surface. (For a demonstration of this, see Ref. 1.) It is indeed from the latter condition, by which the normal component of vorticity is reduced to zero, in combination with the former property that excludes the possibility of the vortex tubes ending on any stationary solid surface.[4] This results in the enlargement, toward the surface, of the cross-sectional area of the vortex tube; the area change, together with the no-slip condition again, causes the variation of the swirl velocity in the direction of the vortex axis. This change in the circumferential velocity sets up, in turn, the pressure gradient along the vortex core, which then induces the cross transport. In addition to the preceding mechanism, which is applicable even to a stationary fluid, the sheared velocity profile within the boundary layer over the sidewalls may further enhance the foregoing effect, in much the same manner as discussed in Ref. 5.

## Acknowledgment

This work was supported by the Air Force Office of Scientific Research under Grant No. AFOSR-83-0049 to the University of Tennessee Space Institute and AFSOR-87-0258 DEF to the University of Washington. Thanks are due to G. Yu (presently at Beijing Institute of Mechanics) for taking the smoke photograph.

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# Interpretation of Jet Mixing Using Fractals

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## Introduction

In Ref. 1, Mandelbrot proposes that constant scalar property surfaces in homogeneous, isotropic turbulence are fractal surfaces and suggests two values for the fractal dimension $D$, depending on the spatial variance of the scalar field: $D = 2\frac{1}{2}$ for Gauss-Burger turbulence and $D = 2\frac{2}{3}$ for Gauss-Kolmogorov turbulence. Lovejoy[2] for clouds and Sreenivasan and Meneveau[3] for shear flows obtain data suggesting fractal dimensions between approximately 2.32 and 2.4 for constant-property surfaces. Hentschel and Procaccia[4] and Kingdon and Ball[5] report analyses of cloud dispersion in homogeneous, isotopic turbulence. In these analyses, molecular transport is ignored, high Reynolds number is assumed, and the dispersion of one species into another is modeled. The structure function $S(l,t)$ for a scalar $\theta$, defined over a volume $V$ as

$$S(l,t) = \frac{1}{V}\int \mathrm{d}r\langle|\theta(r+l,t) - \theta(r,t)|^2\rangle$$

is introduced in the modeling. Both Hentschel and Procaccia and Kingdon and Ball suggest a form for $S(l,t)$ as $|l| \to 0$. Namely, $\lim_{|l|\to 0} S(|l|) \sim |l|^B$. Both analyses contain heuristic elements, and there is a fundamental difference between the two sets of authors in their interpretation of $B$ and its relationship to $D$. Yet their final expressions for $D$ are the same:

$$D = 2 + \frac{2+\mu}{6} \tag{1}$$

where $\mu$ is the intermittency exponent.

In this note, a semi-empirical model of mixing in jet flows is developed that assumes the instantaneous jet structure can be represented by an ensemble of fractal, constant-property surfaces. The model gives an expression for the mean jet fluid concentration on the jet centerline and, for Reynolds number independence, requires that $D = 2 + [(2+\mu)/6]$. This expression for $D$ is the major finding to be reported. While it is identical to that of others, it is obtained by very different reasoning and for a different flow, a free shear flow.

## Jet Mixing Model

For analysis, the following major assumptions are made.

1) The flow is an axisymmetric jet issuing into a stagnant fluid with uniform and constant density.

2) The turbulence is stationary, and the flow is Reynolds number independent.

3) The distributions of mean composition and mean velocity follow similarity.

4) Surfaces of constant, instantaneous jet fluid mass fraction may be represented by fractal surfaces for the purpose of estimating the ensemble mean area of such surfaces.

The instantaneous jet structure can be viewed as a set of constant jet fluid mixture fraction $Z$ surfaces, and it is hypothesized that these surfaces exhibit fractal character over a range of length scales associated with the scales of the turbulent velocity fluctuations, i.e., from the Kolmogorov scale $\eta$ to the integral scale $l$. For a constant $Z$ surface of area $S$ enclosing a volume $V$, an integral equation for jet fluid mass conservation can be written. Then, for stationary turbulence, one can obtain, by taking the ensemble average of this equation, an equation expressing a balance between the mean flux of jet fluid issuing from the jet $M_j$ and the mean jet fluid mass flux across the $Z$ constant surface:

$$M_j = \pi(\mathrm{d}/2)^2\rho V_j = \langle \int_S [\rho Z(\boldsymbol{q} - \boldsymbol{q}_b) + \boldsymbol{j}_z] \cdot \mathrm{d}\boldsymbol{s} \rangle \tag{2}$$

where $V_j$ is the initial volume flux weighted average jet velocity, $d$ is the initial jet diameter, $\boldsymbol{q}$ is the fluid velocity, $\boldsymbol{q}_b$ is the $Z$ surface velocity, and $\boldsymbol{j}_z$ is the diffusive flux of jet fluid across the $Z$ surface. It is assumed that the $Z$ surface is attached to the lip of the jet and that this surface is simply connected. The first assumption is valid, and if the second assumption is relaxed, it seems reasonable to argue that the average flux across the $Z$ surface can still be given by the right-hand side of Eq. (2).

Received May 22, 1987; revision received Jan. 18, 1988. 

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